On the existence of solutions to the stationary Navier-Stokes equations

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Abstract

Existence of a q-weak solution to the stationary Navier-Stokes system in a Lipschitz bounded or exterior domain Ω of E_n, $n = 2, 3$ ($n = 3$ for Ω exterior), is proved under a computable "smallness" condition on the flux of the kinetic field through the boundary of every bounded connected component of $C\bar{\Omega}$ In particular, it is showed that if Ω is a bounded domain of E_3, there exists a positive constant ϵ, depending only on the Lipschitz character of Ω, such that if $a \in W^{1-\frac{1}{q},q}(\partial\Omega)$, with $q \in [2, 3+\epsilon)$ ($q \in [12/7, +\infty)$ if Ω is of class C^l), then the boundary-value problem associated with the Navier-Stokes equations admits at least q-weak solution which, for $q > 2$, assumes the boundary value a in the sense of the nontangential convergence. Moreover, if $a \in W^{1,2}(\partial\Omega)$, then $u \in W^{\frac{3}{2},2}(\Omega)$ and $p \in W^{\frac{1}{2},2}(\Omega)$. The boundary value problem with datum $a \in L^q(\partial\Omega)$ is also considered; it is proved that if $q \geq 8/3$, , then the Navier- Stokes system admits at least one solution which takes the boundary value in a certain sense which reduces to the one of the nontangential convergence for $q > 6$; if $a \in C^{0,\mu}(\partial\Omega)$, with $\mu \in [0, \alpha)$ and $\alpha (< 1)$ depending only on the Lipschitz character of Ω ($\alpha \in (0, 1)$ if Ω is of class C^1), then $u \in C^{0,\mu}(\bar{\Omega})$. The results are obtained by following a classical procedure by J. Leray and performing a careful study of the linear Stokes system.